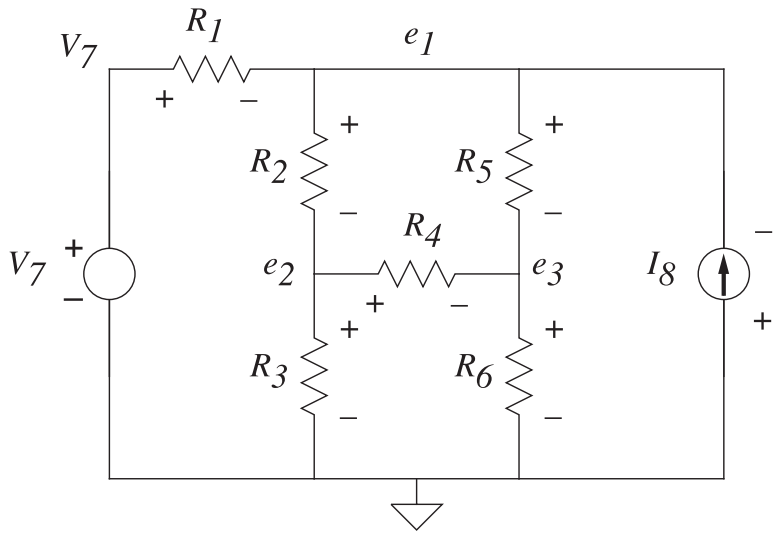
Unified Engineering I **Fall 2006**

Problem S3 (Signals and Systems) SOLUTION (Corrected)



1. To begin, we must label each of the elements of the circuit with a polarity (+/− signs) in order to be able to speak about the element voltages. Note that this labeling is arbitrary, so you may get an answer with different signs. I have labeled the elements as above. I have also assigned a ground node, labeled the one known node (V_7), and labeled the remaining three nodes e_1 , e_2, and e_3. Using the node method, the node equations are then, in order,

$$\begin{aligned}
(G_1+G_2+G_5)e_1 \quad & -G_2e_2 \quad & -G_5e_3 = I_8 + G_1V_7 \text{ (not just } I_8) \\
-G_2e_1 + (G_2+G_3+G_4)e_2 \quad & & -G_4e_3 = 0 \\
-G_5e_1 \quad & -G_4e_2 + (G_4+G_5+G_6)e_3 = 0 &
\end{aligned}$$

Plugging in values, using $G = 1/R$, we have (leaving out the units)

$$\begin{aligned}
3e_1 \quad & -e_2 \quad & -e_3 = 8 \text{ (not 3)} \\
-e_1 + 1.416\overline{6}e_2 \quad & -0.16\overline{6}e_3 = 0 & \\
-e_1 \quad & -0.16\overline{6}e_2 + 1.416\overline{6}e_3 = 0 &
\end{aligned}$$

Solve using a calculator or Matlab or row reduction, we have that

$$\begin{aligned}
e_1 &= \frac{40}{7}\,\text{V} \approx 5.7143\,\text{V} \\
e_2 &= \frac{32}{7}\,\text{V} \approx 4.5714\,\text{V} \\
e_3 &= \frac{32}{7}\,\text{V} \approx 4.5714\,\text{V}
\end{aligned}$$

The element voltages are then found by differencing node potentials:

$$\begin{aligned}
v_1 &= V_7 - e_1 = -\frac{5}{7}\,V \\
v_2 &= e_1 - e_2 = \frac{8}{7}\,V \\
v_3 &= e_2 - 0 = \frac{32}{7}\,V \\
v_4 &= e_2 - e_3 = 0\,V \\
v_5 &= e_1 - e_3 = \frac{8}{7}\,V \\
v_6 &= e_3 - 0 = \frac{32}{7}\,V \\
v_7 &= 5\,V \\
v_8 &= 0 - e_1 = -\frac{40}{7}\,V
\end{aligned}$$

The currents are found from the constitutive relations:

$$\begin{aligned}
i_1 &= \frac{v_1}{R_1} = -\frac{5}{7}\,\mathrm{A} \\
i_2 &= \frac{v_2}{R_2} = \frac{8}{7}\,\mathrm{A} \\
i_3 &= \frac{v_3}{R_3} = \frac{8}{7}\,\mathrm{A} \\
i_4 &= \frac{v_4}{R_4} = \frac{0}{7}\,\mathrm{A} \\
i_5 &= \frac{v_5}{R_5} = \frac{8}{7}\,\mathrm{A} \\
i_6 &= \frac{v_6}{R_6} = \frac{10}{7}\,\mathrm{A} \\
i_8 &= I_8 = 3\,\mathrm{A}
\end{aligned}$$

To find i_7, we must apply KCL at the V_7 node, which implies that $i_7 + i_1 = 0$, so that $i_7 = 5/7\,\mathrm{A}$.

2. The net power is

$$\sum_{n=1}^{8} i_n v_n = \left(\frac{-5}{7}\cdot\frac{-5}{7} + \frac{8}{7}\cdot\frac{8}{7} + \frac{8}{7}\cdot\frac{32}{7} + 0\cdot 0 + \frac{8}{7}\cdot\frac{8}{7} + \frac{8}{7}\cdot\frac{32}{7} + \frac{-5}{7}\cdot 5 + 3\cdot\frac{-40}{7}\right)\mathrm{W} = 0\ \mathrm{W}$$

which is zero by direct calculation.