QM7 – CRIB SHEET

Torsion of round shafts

An internal torque resultant, T generates a circumferential shear stress, τ, at a radius r, and twist per unit length, $\frac{d\phi}{dx}$, where:

$$\tau = \frac{Tr}{J} \qquad\qquad T = GJ\frac{d\phi}{dx}$$

G is the shear modulus of the material and J is the second polar moment of area given by:

$$J = \int_A r^2 dA$$

For a solid circular cross section, radius R: $J = \frac{\pi R^4}{2}$

For a thin walled circular tube, radius R, thickness t: $J = 2\pi R^3 t$

Elastic buckling of columns

The general governing equation for the transverse (buckling) , w, of a uniform column of bending stiffness EI, under an axial load P is: $\frac{d^2w}{dx^2} + \frac{P}{EI}w = M_0$. Where M_0 is a constant.

General solutions are of the form:

$$w = A\sin\left(\sqrt{\frac{P}{EI}}x\right) + B\cos\left(\sqrt{\frac{P}{EI}}x\right) + Cx + D.$$

In general the elastic critical load, $P_{cr} = cP_E$, where the factor c depends on the boundary conditions and the order of the buckling mode, and P_E is the Euler Load for a perfect, pin ended column of length, L buckling into a half sine wave given by:

$$P_E = \frac{\pi^2 EI}{L^2}$$

Yield and Plasticity of Metals

Uniaxial loading of a bar, initial length ℓ_0, cross-sectional area A_0 past yield point: Define nominal, true stress and nominal and true strain:

$$\sigma_n = \frac{P}{A_0}, \qquad \sigma_t = \frac{P}{A}, \qquad \varepsilon_n = \frac{\Delta\ell}{\ell_0} = \frac{\ell - \ell_0}{\ell_0}, \qquad \varepsilon_t = \int_{\ell_0}^{\ell} \frac{d\ell}{\ell} = \ln\left(\frac{\ell}{\ell_0}\right)$$

Since volume is conserved: $A_0\ell_0 = A\ell$ obtain: $\sigma_t = \sigma_n(1+\varepsilon_n)$ and $\varepsilon_t = \ln(1+\varepsilon_n)$

Work of deformation per unit volume: $U = \int_{\varepsilon_{n1}}^{\varepsilon_{n2}} \sigma_n d\varepsilon_n = \int_{\varepsilon_{t1}}^{\varepsilon_{t2}} \sigma_t d\varepsilon_t$

Elastic Strain Energy (for linear elastic deformation): $U = \frac{\sigma_n^2}{2E}$

For multiaxial stress states models for yield:

Tresca: $\max\{|\sigma_I - \sigma_{II}|, |\sigma_{II} - \sigma_{III}|, |\sigma_{III} - \sigma_I|\} \geq \sigma_y$

Von Mises: $(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2 \geq 2\sigma_y^2$

Where σ_I etc are the principal stresses and σ_y is the uniaxial yield strength

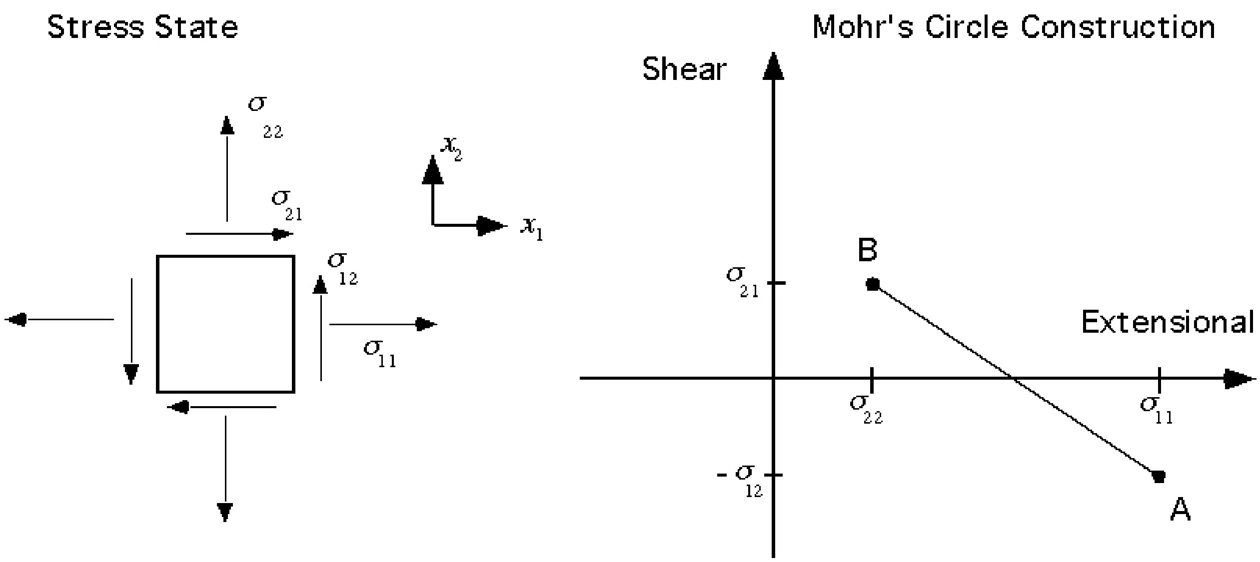
Transformation of Stress and Strain via Mohr's Circle:

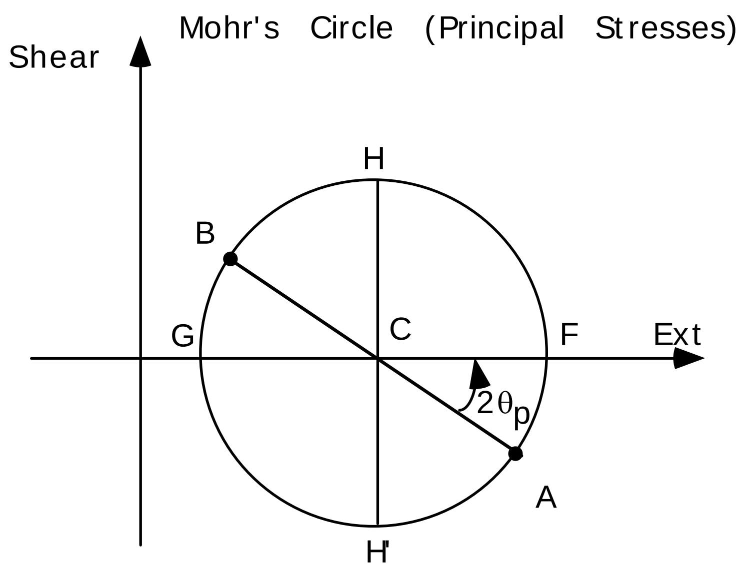
Mohr's circle is a geometric representation of the 2-D transformation of stresses.

Construction: Given the state of stress shown below for an infinitessimal element, with the following definition (by Mohr) of positive and negative shear:

"Positive shear would cause a clockwise rotation of the element about the element center."

Thus: σ_{21} (*below*) is plotted positive σ_{12} (*below*) is plotted negative:





Principal stresses correspond to points G, F. Max shear at H, H'.

Note that angles are doubled on the Mohr's circle relative to the physical problem.
Note that a Mohr's circle can only be drawn stresses in a plane perpendicular to a principal direction.

Fracture and Fatigue

Fast fracture occurs when: $dW \geq dU^{el} + G_c dA$
where W = external work, U_{el} = elastic strain energy, G_c is the material's toughness and A is the area of crack surface.

Can also be written: $K \geq K_c$ Where K_c is the fracture toughness and K is the stress intensity factor given by:

$$K = Y\sigma\sqrt{\pi a}$$

where Y is a factor which depends on the crack and component shape (≈1), a is the crack length and σ the applied stress

For many metals fatigue crack growth is of the form: $\frac{da}{dN} = A\Delta K^n$

where A and n are empirically determined constants.