M19

a) Results from M18: Assume crack can potentially be oriented with maximum principal stress (worst−case scenario)

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In[14]:= t =.; σI = Assuming[t > 0, Simplify[σI]]
```

Out[14]= $\frac{N_x + N_y + \sqrt{N_x^2 - 2 N_x N_y + N_y^2 + 4 S_{xy}^2}}{2 t}$

for the three different loading cases this gives:

```
In[20]:= σI /. loads // N
```

Out[20]= $\left\{\frac{247214.}{t}, \frac{777200.}{t}, \frac{700000.}{t}\right\}$

So the maximum principal stress is $\frac{777.2\,\mathrm{kN}/m}{t}$

This will produce a stress intensity factor K = 1 $\sigma\sqrt{\pi a}$, (where "a" is the critical crack size a=0.01 m),

which should be smaller than the fracture toughness $K_C = 44\,\mathrm{MPa}\sqrt{m}$.Or:

$\frac{777.2\,10^3\,N/m}{t}\sqrt{\pi\,0.01\,m} < 44\,10^6\,\mathrm{Pa}\sqrt{m}$

From where one can obtain "t" to avoid fast fracture

```
In[37]:= Solve[777200/t Sqrt[π 0.01] == 44 10^6, t] // Flatten
```

Out[37]= {t → 0.0031308}

This value is larger than t=2.46 mm, which was the requirement for strength. The fracture criterion prevails

b) The general population should not be concerned. You should explain that aircraft are designed with a "damage tolerance approach" by which it is accepted that structures do have cracks, we just need to know how to deal with them. In other words, we make sure that the design is robust enough to handle cracks. The main idea is to make sure that cracks long enough to cause failure can be detected

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This is M18

In[1]:=
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σ̄ = (σ11  σ12
     σ12  σ22);
principalσ = Eigenvalues[σ̄]
```

Out[2]= $\left\{\frac{1}{2}\left(\sigma_{11}+\sigma_{22}-\sqrt{\sigma_{11}^2+4\,\sigma_{12}^2-2\,\sigma_{11}\,\sigma_{22}+\sigma_{22}^2}\right),\ \frac{1}{2}\left(\sigma_{11}+\sigma_{22}+\sqrt{\sigma_{11}^2+4\,\sigma_{12}^2-2\,\sigma_{11}\,\sigma_{22}+\sigma_{22}^2}\right)\right\}$

In[3]:=
```
σI = principalσ[[2]]
σII = principalσ[[1]]
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Out[3]= $\frac{1}{2}\left(\sigma_{11}+\sigma_{22}+\sqrt{\sigma_{11}^2+4\,\sigma_{12}^2-2\,\sigma_{11}\,\sigma_{22}+\sigma_{22}^2}\right)$

Out[4]= $\frac{1}{2}\left(\sigma_{11}+\sigma_{22}-\sqrt{\sigma_{11}^2+4\,\sigma_{12}^2-2\,\sigma_{11}\,\sigma_{22}+\sigma_{22}^2}\right)$

In[5]:=
```
σIII = 0;
vmFunction = (σI - σII)^2 + (σII - σIII)^2 + (σIII - σI)^2 - 2 σy^2 == 0 // Simplify
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Out[6]= $\sigma_{11}^2+3\,\sigma_{12}^2+\sigma_{22}^2==\sigma_{11}\,\sigma_{22}+\sigma_y^2$

Express the stress components in terms of the line forces and use a safety factor of 1.5:

In[7]:=
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σ11 = Nx/t; σ12 = Sxy/t; σ22 = Ny/t;
vmf = vmFunction /. t → Sqrt[t2]
sol = Solve[vmf , t2] // Flatten
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Out[8]= $\frac{N_x^2}{t2}+\frac{N_y^2}{t2}+\frac{3\,S_{xy}^2}{t2}==\frac{N_x\,N_y}{t2}+\sigma_y^2$

Out[9]= $\left\{t2\to\frac{N_x^2-N_x\,N_y+N_y^2+3\,S_{xy}^2}{\sigma_y^2}\right\}$

Apply safete factor =1.5 to yield stress

In[10]:=
```
sol2 = sol /. σy → σy/1.5 // Simplify;
t = Sqrt[t2] /. sol2
```

Out[11]= $1.5\sqrt{\frac{N_x^2 - N_x N_y + N_y^2 + 3 S_{xy}^2}{\sigma_y^2}}$

In[12]:=
```
loads = {
   {Nx → 200 10^3, Ny → -600 10^3, Sxy → 200 10^3},
   {Nx → 200 10^3, Ny → 500 10^3, Sxy → -400 10^3},
   {Nx → 700 10^3, Ny → -100 10^3, Sxy → 0}
  };
Table[t /. loads[[i]] /. σy → 500 10^6, {i, 3}]
```

Out[13]= {0.0024, 0.00245561, 0.00226495}

Of the three load cases, the second one furnishes the thickest skin t=2.46 mm and should be the minimum thickness used.