Review on Summations

- **Constant Series:** For integers a and b, $a \le b$,

$$\sum_{i=a}^{b} 1 = b - a + 1$$

- **Linear Series (Arithmetic Series):** For $n \ge 0$,

$$\sum_{i=1}^{n} i = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

- **Quadratic Series:** For $n \ge 0$,

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Review on Summations

- **Cubic Series:** For $n \ge 0$,

$$\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

- **Geometric Series:** For real $x \ne 1$,

$$\sum_{k=0}^{n} x^k = 1 + x + x^2 + \cdots + x^n = \frac{x^{n+1} - 1}{x - 1}$$

For $|x| < 1$,

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$$

Review on Summations

- **Linear-Geometric Series:** For $n \ge 0$, real $c \ne 1$,

$$\sum_{i=1}^{n} ic^i = c + 2c^2 + \cdots + nc^n = \frac{(n-1)c^{n+1} - nc^n + c}{(c-1)^2}$$

- **Harmonic Series:** nth harmonic number, $n \in \mathrm{I}^+$,

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$

$$= \sum_{k=1}^{n} \frac{1}{k} = \ln(n) + O(1)$$

Review on Summations

- **Telescoping Series:**

$$\sum_{k=1}^{n} a_k - a_{k-1} = a_n - a_0$$

- **Differentiating Series:** For $|x| < 1$,

$$\sum_{k=0}^{\infty} kx^k = \frac{x}{(1-x)^2}$$

Logarithms

$$a = b^{\log_b a}$$

$$\log_c(ab) = \log_c a + \log_c b$$

$$\log_b a^n = n \log_b a$$

$$\log_b a = \frac{\log_c a}{\log_c b}$$

$$\log_b(1/a) = -\log_a b$$

$$\log_b a = \frac{1}{\log_a b}$$

$$a^{\log_b c} = c^{\log_b a}$$

$$a^{-1} = \frac{1}{a}$$

$$(a^m)^n = a^{mn}$$

$$a^m a^n = a^{m+n}$$

$\log_b(x/y) = \log_b x - \log_b y$

$a^{(b+c)} = a^b a^c$

$a^b / a^c = a^{(b-c)}$

$b = a^{\log_a b}$

$b^c = a^{c \log_a b}$

Simplified Master Method

T(n) = aT(n/b) + cn^k,

where a,c > 0 and b > 1

$$\textbf{T(n)} = \begin{cases} O\left(n^{\log_b a}\right) & a > b^k \\ O\left(n^k \log_b n\right) & a = b^k \\ O\left(n^k\right) & a < b^k \end{cases}$$