Development Economics
Fall 2003
MIT 14.771; Harvard Economics 2390B
Problem Set 1

The due date for this problem set is Monday, October 6th, 2003.
Please e-mail any questions to:
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1. This question asks you to work through parts of the model used by Klenow & Rodriguez-Clare (1997). Klenow & Rodriguez-Clare discuss the model presented by Mankiw, Romer & Weil (1992), who incorporated human capital into the Solow (1956) model.

   The assumptions of this model are:

$$Y = K^{\alpha} H^{\beta} (AL)^{1-\alpha-\beta}$$
$$\dot{L} = nL$$
$$\dot{A} = gA$$
$$\dot{K} = I_K - \delta K$$
$$\dot{H} = I_H - \delta H$$
$$I_K = s_K Y$$
$$I_H = s_H Y$$

   Where Y denotes output, K denotes physical capital, H denotes human capital, A denotes productivity, L denotes labor, I_K denotes the gross investment in physical capital and I_H denotes the gross investment in human capital. The assumptions about the constants are: $\alpha, \beta, \delta, g, n, s_K, s_H \in (0,1)\,;\ \alpha + \beta < 1$.

   Define:

$$k = K/(AL)$$
$$h = H/(AL)$$
$$y = Y/(AL)$$

   As physical capital per effective unit of labor, human capital per effective unit of labor and output per effective unit of labor, respectively.

   (a) Derive the expressions for $\dot{k}$ and $\dot{h}$

   (b) Derive the steady state values k^* and h^*

   (c) Calculate $\ln(y^*)$

   (d) Prove that in steady state: $K/Y = \frac{I_K/Y}{g+\delta+n}$, $H/Y = \frac{I_H/Y}{g+\delta+n}$

2. Consider the following model of technology adoption:

Nature randomly determines whether the best decision for people is to *adopt* a new technology ($V = 1$) or to *reject* it ($V = 0$). The probability of each outcome is 1/2.

Subsequently, a sequence of individuals has to decide whether or not to adopt the new technology. Each player takes one action: *adopt* or *reject.*

Each individual who makes the correct decision (the same decision as nature) gets a payoff of 1, regardless of the payoffs to others; an individual who makes the wrong decision gets a payoff of 0.

Before making her decision, each individual i observes a signal X_i that is independent of the signals of all the other players and can take on the value A or R with the following probabilities:

	$\Pr(X_i = A \mid V)$	$\Pr(X_i = R \mid V)$
$V = 1$	p	$1 - p$
$V = 0$	$1 - p$	p

Where $p > 1/2$.

All players have a flat prior about V - that is, they understand that V=1 and V=0 can happen with equal probabilities.

Each player forms a posterior belief about V, taking into account her own signal and the actions of all those that moved before her. As a tie breaking convention, assume that an individual indifferent between adoption and rejection adopts or rejects with an equal probability of 1/2.

(a) How does the move of the first player depend on her signal?

(b) How does the second player decide to move? Assume that *adopt* is the high payoff action ($V = 1$). Conditional on $V = 1$, what is the probability that the first two players choose (*adopt, adopt*)? That they choose (*reject, reject*)? That the first two moves are different?

(c) Given that the first two moves were (*adopt, adopt*), what does player three choose if her signal is $X_3 = A$? If her signal is $X_3 = R$? Explicitly compute the posterior likelihood of (*adopt, adopt,* $X_3 = A$) given $V = 1$ and $V = 0$. Given that the first two moves were (*reject, reject*), what does player three choose if her signal is $X_3 = A$? If her signal is $X_3 = R$? How likely is inefficient herding after the first two moves as a function of p? Give the intuition.

(d) What is the probability that no herd has formed after 4 players have moved? After n players have moved (n even)?